preprint - UTM 656/UTF 453

Central Charges and Boundary Fields for Two Dimensional Dilatonic Black Holes

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December 2003

Abstract: *In this paper we first show that within the Hamiltonian description of general relativity, the central charge of a near horizon asymptotic symmetry group is zero, and therefore that the entropy of the system cannot be estimated using Cardy's formula. This is done by mapping a static black hole to a two dimensional space. We explain how such a charge can only appear to a static observer who chooses to stay permanently outside the black hole. Then an alternative argument is given for the presence of a universal central charge. Finally we suggest an effective quantum theory on the horizon that is compatible with the thermodynamics behaviour of the black hole.*

PACS numbers: *04.20.-q, 04.70.-s, 04.70.Dy*

1 Introduction

Following the discovery that black holes behave like thermodynamic systems (see [1] for a review with an exhaustive list of references), many efforts have been done to unravel the structure of the underlying microscopic theory. Most of these focused on the event horizon of the black hole as the place where interesting new physics is possibly operating, and in fact the ability to obtain black hole thermodynamics from horizon quantum states has always been considered as a strong test to the viability of any model of quantum gravity. Entropy computations in particular were one of the main activity for people working in string theory and quantum gravity.

But as has became clear over the years, the horizons associated to black holes are not the only ones that have thermodynamics. The cosmological horizons, the distance light can travel between now and the end of time, also have the ability to exchange information with the regions they trap (classically they would not), and to create a noisy background radiation with a characteristic thermal spectrum [2]. But in view of the structureless nature of the cosmological horizons the counting of the corresponding microstates is even more acute than for black holes (see [3] for a recent review). Nevertheless it has been shown [4] that the validity of the generalized second law would be seriously challenged were it not for the geometric entropy of the cosmological horizon.

Perhaps one may notice, at this point, that the entropy we are talking about is of order $\hbar^{-1}$ and is finite, whereas the one-loop partition function from field theory, the one from which the entropy could be derived, is of order $\hbar^0$ and ultraviolet divergent[1] [5]. This indicates that back reaction effects must be important and also that the black hole spectrum must be strictly discrete, with a quantized area.
Bekenstein [6] [7] first argued that if the horizon area were quantized, whichever that means, its eigenvalues would be equally spaced because the area is an adiabatic invariant. In this picture the area is a sum of cells with the same number of degrees of freedom per cell. Since then, a number of authors [8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22] have given arguments for an equally spaced area spectrum. Moreover, it has been observed [23, 24, 25] that the spacing coefficient can be fixed using the spectrum of the quasi-normal modes of the Schwarzschild's black hole. Even earlier than this, J. York [26] proposed to quantize the quasi-normal modes of a non rotating black hole; using a "common sense" quantization of the modes he obtained results surprisingly close to the (in)famous $A/4$.
Another approach was to describe the invisible hairs by means of quantum fields around the black hole. In principle there is nothing exotic about this picture, but the entropy of these fields was found to be quadratically divergent as a function of the (inverse) distance from the horizon [5]. 't Hooft observation was that regularizing with a cutoff of the order of the Planck length, gives an entropy satisfying the area law and with the right magnitude to be identified with the entropy of the black hole. Notice that without divergences this would not be possible. The thermal entropy of quantum fields around the black hole can also be understood as entropy of entanglement [27], since a static observer hanging over the black hole would have to trace over the hidden degrees of freedom. The fact that a cutoff like the Planck scale must be invoked in an otherwise conventional theory, indicates the necessity to take into account quantum gravity effects[2].
Still another idea was to describe the microscopy of black holes by counting the number of states of a conformal field theory living on the boundary of spacetime. Many years ago J. Brown and M. Henneaux [30] made this observation by counting the dimension of the asymptotic symmetry group at conformal infinity in anti-de Sitter space. It was later realized by Strominger [31] that this counting gave complete agreement with the Bekenstein-Hawking entropy of the BTZ black hole [32]. Today we understand this success as a particular case of the $\mathrm{AdS}_{d+1}/\mathrm{CFT}_d$ correspondence [33], which describes gravity by a dual conformal field theory on the boundary at infinity of the conformal compactification of anti-de Sitter space.
Moreover some supersymmetric extremal and near-extremal black holes configurations consisting of D-brane matter were also accounted for by a conformal field theory living on the branes [34, 35, 36], so in this case the boundary was near the horizon rather than at infinity.
Therefore people tried to obtain a similar performance for the near horizon symmetry of non extreme, non supersymmetric configurations [37, 38, 39], but these treatments seemed to have some flaw [40,41,42]. In particular it has been shown that after fixing the near horizon symmetry compatibly with certain boundary conditions, the central charge of the putative CFT vanishes [43, 44].
In this paper we shall show, rigorously, that the central charge of the near horizon symmetry of a two dimensional black hole vanishes for asymptotic symmetries that preserve the surface gravity or the horizon area. We stress that such a charge can only arise for an observer who chooses to stay outside the black hole and has an event horizon, because such an observer must use an Hamiltonian associated to a singular space-time foliation.

[1] We use this term to emphasize the fact that the divergence is not associated with the infinite volume of the space.

[2] This has been stressed particularly lucidly, in our opinion, by 't Hooft [5], Jacobson [28], and Susskind-Uglum [29].

As in the well known example in three dimensions, the full group of diffeomorphisms in the presence of a boundary is not a symmetry group of the theory, but we expect that this symmetry can be recovered in the full microscopic description. Then we shall construct a covariant action for the theory of gravity in two dimensions with boundary. For this purpose we add some ad hoc fields living on the horizon. Then fixing the energy (the mass of the black hole) we argue that these fields describe the whole black hole (an uncharged one). We show that if we count the degrees of freedom of this gauge invariant theory on the boundary, we recover the value of the semiclassical entropy. The remainder of the paper is organized as follows: In Sec. [2] we show how a two dimensional black hole with dilaton can be obtained from dimensional reduction of an higher dimensional one with spherical symmetry, and why such a theory may explain the entropy of the parent, original black hole. In Sec. [3] we present a detailed description of the Hamiltonian generators of a two dimensional theory with dilaton, from the viewpoint of an observer who chooses to stay permanently outside a given black hole. In Sec. [4] we treat the horizon as an inner boundary and show that the Hamiltonian generators corresponding to timelike deformations form an abelian algebra with no central extension. In sec. [5] we give an informal argument suggesting that a central charge must be present anyway. In sec. [6] we build an action covariant with respect to every diffeomorphism, by adding some ad hoc boundary fields. Then we add some observations concerning quantization, the density of states and the entropy of this system.

2 Two dimensional models

To investigate some of the properties that make black holes so special, it is useful to study the two dimensional effective theories that can be obtained by dimensional reduction from higher dimensional ones. Many interesting scalar-tensor two dimensional theories can be obtained in this way. One important class of theories with just one scalar and the metric, emerges from spherically symmetric reduction. A much studied example is the Jackiw-Teitelboim model [45] (shortened JT therefrom) which can be obtained by dimensional reduction from three-dimensional anti-de Sitter space AdS_3. Theories in which one or more scalars couple to gravity in two dimensions fall under the generic name of "dilaton gravity" [46]. It must be recognized, however, that using these models in black hole physics one neglects all transverse excitations of the black hole, and therefore works with an approximate scheme that may simply be wrong (see [47] for an alternative approach).
We are interested in studying the reduction of the Schwarzschild's black hole, so we start with a metric of the form (we take the signature "mostly plus")

$$ds^2 = \gamma_{ab}dx^a dx^b + X^2(x^a)d\omega_{d-2}^2 \tag{2.1}$$

where $d\omega_{d-2}^2$ is the line element of a compact space with volume ω_{d-2}. Up to a total derivative the Einstein-Hilbert action is

$$\begin{aligned} I_d \;=\; & \frac{\omega_{d-2}}{16\pi G_d}\int_\Sigma \Big((d-2)(d-3)X^{d-4}\gamma^{ab}\partial_a X\partial_b X + X^{d-2}\mathcal{R}[\gamma] \\ & + \kappa(d-2)(d-3)X^{d-4} - 2\Lambda X^{d-2}\Big)\,|\gamma|^{1/2}d^2x \end{aligned} \tag{2.2}$$

where $\kappa = 0, \pm 1$ is the curvature of $d\omega_{d-2}^2$ and Σ is a two-surface. In three dimensions we get the JT model; taking $\omega_1 = 2\pi$ the action is

$$I = \frac{1}{8G}\int X(\mathcal{R} - 2\Lambda)|\gamma|^{1/2}d^2x$$

In four dimensions the action (2.2) is quadratic in the field[3] X

$$I_4 = \frac{\omega_2}{8\pi G_4}\int_\Sigma |\gamma|^{1/2}\left(\gamma^{ab}\partial_a X\partial_b X + \frac{1}{2}X^2\mathcal{R}[\gamma] + \kappa - \Lambda X^2\right) d^2x \tag{2.3}$$

and leads to the following field equations

$$-\Box_\gamma X + X\mathcal{R}[\gamma] - 2\Lambda X = 0 \tag{2.4}$$

$$T_{ab} \equiv \partial_a X\partial_b X - \frac{1}{2}\gamma_{ab}\partial_c X\partial^c X - \frac{1}{2}\nabla_a\partial_b(X^2) + \frac{1}{2}\gamma_{ab}\Box_\gamma(X^2) + \frac{1}{2}(\Lambda X^2 - \kappa)\gamma_{ab} = 0 \tag{2.5}$$

The kinetic term of the X field in (2.3) is that of a spacelike string coordinate, so one is tempted to think that X is not a physical field. However, when the curvature coupling is non zero either sign for the kinetic energy is legitimate, since the X field mixes with the conformal factor of the metric in such a way that the signature of the resulting kinetic matrix is $(+,-)$. In fact, by an appropriate Weyl rescaling, $\gamma_{ab} = X^{-2}q_{ab}$, one obtains the action (we set $\kappa = 1$ and $\omega_2 = 4\pi$ as is appropriate to spherical symmetry)

$$I_4 = -\frac{1}{2G}\int_\Sigma |q|^{1/2}\left(q^{ab}\partial_a X\partial_b X - 2^{-1}X^2\mathcal{R}[q] - X^{-2} + \Lambda\right) d^2x$$

This has now a standard kinetic term but a non trivial potential. Both theories are just a version of dilaton gravity [46, 48, 49, 50], so are equivalent to a $c = 26$ conformal sigma model with $2D$ target space[4]. The field equations for the action (2.3) in a flat conformal gauge with $\gamma_{ab} = \eta_{ab}\exp(2\phi)$ and coordinates $(t, x) \in \mathbb{R}^2$, are

$$\Box_\eta X + 2X\Box_\eta\phi + 2\Lambda X e^{2\phi} = 0 \tag{2.6a}$$

$$X\Box_\eta X + \eta^{ab}\partial_a X\partial_b X = (\kappa - \Lambda X^2)e^{2\phi} \tag{2.6b}$$

plus two constraints (i.e. the missing equations of motion $T_{ab} = 0$ written with respect to null coordinates $x_\pm = t \pm x$)

$$T_{\pm\pm} = -X\left(\partial_\pm^2 X + 2\,\partial_\pm\phi\partial_\pm X\right) = 0 \tag{2.7}$$

The remaining equation, $T_{+-} = 0$, is equivalent to (2.6b). Some solutions are: for $\Lambda = 0$ the trivial, flat solution $\kappa = 1$, $X = x$, $\phi = 0$ and the non trivial Schwarzschild's solution with $\kappa = 1$ and mass parameter $M = 2a$

$$x = X + a\ln(X/a - 1), \qquad e^{2\phi} = 1 - a/X$$

We see that x is the well known tortoise coordinate for the Schwarzschild black hole, while the dilaton X is a non trivial function of x. For $\Lambda > 0$ and $\kappa = 1$ there is a constant solution $X = 1/\sqrt{\Lambda}$, $R[\gamma] = 2\Lambda$, namely the Nariai extreme black hole $dS_2 \times S^2$ with product metric. For $\Lambda < 0$ and $\kappa = -1$ there is a Bertotti-Robinson solution $AdS_2 \times S^2$ with product metric, and of course there will be many other solutions with varying X.

[3] Viewed as a field in two dimensions, X does not have canonical dimension.
[4] With coordinates (X, ϕ), ϕ the Liouville field.

Conversely, given a solution of two dimensional dilaton gravity we can form the metric (2.1) and this will solve Einstein equations in higher dimensions.
The kinetic term can also be made to disappear, by using the Weyl rescaling $\gamma^{ab} \to X\gamma^{ab}$ and a field reparametrization $\eta = X^2$ at the same time. Then the action takes the form

$$I = \frac{1}{4G}\int_\Sigma |\gamma|^{1/2}\left(\eta\mathcal{R}[\gamma] + 2V(\eta)\right) d^2x \tag{2.8a}$$

$$V(\eta) = \frac{1}{\sqrt{\eta}} - \Lambda\sqrt{\eta} \tag{2.8b}$$

and this is the action we will use. From (2.6) we see that the dilaton mixes non trivially with the Liouville field. In principle, the effective action governing the X, or η, fields, can be obtained by integrating $\exp iI$ over all $2D$ metrics on Σ modulo all diffeomorphisms. It is this last integration that produces the $c = -26$ anomaly that in dilaton gravity is supposed to be cancelled by the other fields.
So we have to study a classical two dimensional dilaton theory. In the case of dimensional reduction from a four dimensional theory, $V(\eta) = 1/\sqrt{\eta} - \Lambda\sqrt{\eta}$, but we shall study more general actions with arbitrary potentials. For example, with a linear dilaton potential one has the JT model [45], and with a constant potential the CGHS model [51]. The equations of motion are:

$$\mathcal{R}[\gamma] + 2\partial_\eta V(\eta) = 0, \qquad \gamma_{ab}\Box\eta - \nabla_a\nabla_b\eta = V(\eta)\gamma_{ab} \tag{2.9}$$

Note that the value of the dilaton on the horizon of the parent, higher dimensional black hole, is the area of the horizon spatial section divided by 4π. In this way the $2D$ theory remembers its higher dimensional origin.
This is a good place to mention an important consistency condition, which is that the entropy of the original higher dimensional black hole must match correctly with the entropy of the two dimensional theory. In higher dimensions and within general relativity (i.e. with the Einstein-Hilbert action determining dynamics) the entropy is given by the usual Bekenstein-Hawking area law, $S = A/4G$. In two dimensions we do not have horizons with finite area but we have a valid substitute, the Noether charge [52]. Given a stationary black hole solution in the theory (2.8), with Killing vector field ξ^a and surface gravity $\kappa = -2^{-1}\varepsilon^{ab}\nabla_a\xi_b$, one can define a conserved charge on the horizon that is quite independent on the form of the potential. This Noether charge 0-form[5] turns out to be

$$Q_H = \kappa\eta_0/2G = \kappa A/8\pi G \tag{2.10}$$

where $\eta_0 = A/4\pi$ is the dilaton restricted on the horizon. Given the charge, the entropy is [53]

$$S = 2\pi\kappa^{-1}Q_H = \pi G^{-1}\eta_0 = A/4G$$

as we wanted to show. Even if there were a mathematical explanation of this coincidence it remains a little bit surprising. In the event that a state counting in $2D$ dilaton gravity succeeds in computing the Noether charge, that would be a successful state counting for the entropy of almost all higher dimensional, non rotating black holes!
In the following, we shall investigate the Hamiltonian structure of these generic dilatonic theories[6].

[5]The Noether charge is a $(d-2)$-form in d dimensions.
[6]A rather general treatment of phase space formulation of $2D$ gravity models can be found in [54]. The quantum theory is studied in [46]. See also [55, 56, 57] for extensive reviews on the subject.

3 The Hamiltonian generators and the central charge

In the first part of this section we briefly summarize the classical Hamiltonian for two dimensional dilaton gravity. Then we shall directly compute the Poisson bracket of the Hamiltonian generators in the presence of boundary terms. This is inspired by the work of Brown, Lau and York [58], who computed the Poisson brackets of the Hamiltonian generators with boundary terms, in four dimensions. The ADM form of a two dimensional metric is

$$ds^2 = -N^2 dt^2 + \sigma^2 (dx + N^x dt)^2 \tag{3.1}$$

and we take the action in the form given by (2.8) , with a general potential. Discarding for the moment the boundary terms, the Lagrangian density $\mathcal{L}$ becomes (we set $2G = 1$ for the time being and restore Newton constant afterwards)

$$\mathcal{L} = -\frac{\dot{\eta}\dot{\sigma}}{N} - N\left(\frac{\eta'}{\sigma}\right)' + \sigma N V(\eta) - \frac{N^x (N^x\sigma)'\eta'}{N} + \frac{\dot{\sigma}N^x\eta'}{N} + \frac{\dot{\eta}(\sigma N^x)'}{N} \tag{3.2}$$

where we agree that one or more primes will denote spatial derivatives (i.e. derivatives with respect to x) and one or more overdots will indicate time derivatives. If we want a Hamiltonian formulation we also need to find the conjugate momenta Π_η, Π_σ and Π_N, which are

$$\Pi_\eta = \frac{-\dot{\sigma} + (N^x\sigma)'}{N}, \qquad \Pi_\sigma = \frac{-\dot{\eta} + N^x\eta'}{N}, \qquad \Pi_N = 0 \tag{3.3}$$

The bulk Hamiltonian will be a sum of constraints

$$H = \int dx (N\mathcal{H}_\perp + N^x \mathcal{H}_x) \tag{3.4}$$

respectively given by

$$\mathcal{H}_\perp = -\Pi_\eta \Pi_\sigma + \left(\frac{\eta'}{\sigma}\right)' - V(\eta)\sigma \sim 0, \qquad \mathcal{H}_x = \Pi_\eta \eta' - \sigma \Pi_\sigma' \sim 0 \tag{3.5}$$

The constraint functionals can also be smeared with test fields ξ^μ and used as generators of the gauge symmetry, which in our case is the diffeomorphism symmetry generated by the vector field ξ^μ.

To obtain well defined generators the test fields ξ^μ, and possibly some first order derivatives as well, must vanish on the boundary. So we define the smeared Hamiltonian constraints

$$H[N^x] = \int_\Sigma dx N^x \mathcal{H}_x, \qquad H[N] = \int_\Sigma dx N \mathcal{H}_\perp \tag{3.6}$$

and $H = H[N, N^x] = H[N] + H[N^x]$. There are terms in the Hamiltonian giving origin to boundary terms in the course of variations. Looking at (3.5), these terms are $N\left(\frac{\eta'}{\sigma}\right)'$ and $N^x\left(\Pi_\eta \eta' - \sigma \Pi_\sigma'\right)$. So, collecting the variations

$$\delta H = \left[N\frac{\delta\eta'}{\sigma} - N\frac{\eta'}{\sigma^2}\delta\sigma - N'\frac{\delta\eta}{\sigma} + N^x \Pi_\eta \delta\eta - N^x \sigma \delta\Pi_\sigma\right]_{\partial\Sigma} + \dots \tag{3.7}$$

the dots denoting bulk terms giving the equations of motion. This expression shows that the action functional is not differentiable if $N \neq 0$ or $\sigma^{-1}N' \neq 0$ or $N^x \neq 0$ on the boundary. To restore differentiability we have to add suitable boundary terms to the Hamiltonian generators [59], but these terms will depend on the choice of boundary conditions, i.e. on the physics we

are going to describe, and the physics we have in mind is the one describing the presence of a static black hole from the viewpoint of an observer who chooses to stay permanently outside it. In that case the boundary term (3.7) has an inner contribution at the horizon, and therefore we will need to impose (rather specific) boundary conditions on the metric near the horizon.
First, the $U(1)$ isometry corresponding to time translations of the metric has a fixed point set at the horizon position x_0, so $N(x_0) = 0$. Second, experience with black holes requires that the geometry of the metric (3.1) be isometric to a flat disk[7]. Denoting κ the surface gravity of the black hole, the Euler characteristic of the metric (3.1) is found to be $\chi = (\kappa\sigma)^{-1}N'$, so requiring $\chi = 1$ gives the regularity condition

$$(\sigma^{-1}N')_{|x=x_0} = \kappa \tag{3.8}$$

For extreme states in particular (solutions with $\kappa = 0$), (3.8) is replaced with $N'(x_0) = 0$, because in this case the black hole has the topology of the annulus.
Finally, it was noted in [60] that the Euler characteristic of the full black hole metric must be two[8], which gives the third condition $\sigma^{-1}(x_0) = 0$.
For the metric (3.1) we summarize these conditions in the form

$$N(x_0) = 0 \tag{3.9a}$$
$$\sigma(x_0)^{-1} = 0 \tag{3.9b}$$
$$(\sigma^{-1}N')_{|x=x_0} = \kappa, \quad \text{a fixed non zero constant} \tag{3.9c}$$

The modified Hamiltonian generators which are differentiables under the given boundary conditions can be obtained by looking to Eq. (3.7), and are[9]

$$H[N^x] = \int_\Sigma dx N^x \mathcal{H}_x - (N^x (\Pi_\eta \eta - \Pi_\sigma \sigma))|_{\partial\Sigma} \tag{3.10a}$$
$$H[N] = \int_\Sigma dx N\mathcal{H} - N\frac{\eta'}{\sigma}\bigg|_{\partial\Sigma} + N'\frac{\eta}{\sigma}\bigg|_{\partial\Sigma} \tag{3.10b}$$

where $\partial\Sigma$ is the surface $x = x_0$ (a point). One can verify that the variations on the surface $x = x_0$ are all zero. The modified Hamiltonian generators are in fact observables because they commute with the equations of motion as well as the constraints [61].
Now that we have a well defined Hamiltonian system, we may evaluate $\dot{\Pi}_\sigma$ and $\dot{\Pi}_\eta$

$$\dot{\Pi}_\sigma = \Pi'_\sigma N^x - \frac{\eta' N'}{\sigma^2} + V(\eta)N, \quad \dot{\Pi}_\eta = (\Pi_\eta N^x)' - \left(\frac{N'}{\sigma}\right)' + \sigma N \frac{\partial V(\eta)}{\partial \eta} \tag{3.11}$$

Together with (3.3), these are equivalent to the original Lagrangian field equations.
The symmetries of the system associated with the vector ξ^μ are generated by the functionals $\mathcal{O}(\xi)$, where

$$\mathcal{O}[\xi] = \int dx \left(\xi^\perp \mathcal{H} + \xi^\parallel \mathcal{H}_x\right) + Q[\xi] \tag{3.12}$$

[7]More correctly, this is true after a Wick rotation $\tau = -it$. In a Lorentzian world, the isometry is with a flat two-dimensional Rindler space. For extreme states, the isometry is with two-dimensional anti-de Sitter space.
[8]Because $\chi(blackhole) = \chi(disk) \times \chi(sphere)$ and $\chi(sphere) = 2$.
[9]We omit the terms at infinity which are familiar and well known.

and $\xi^\perp = N\xi^t$, $\xi^\| = \xi^x + N^x\xi^t$ and $Q[\xi]$ are the boundary term introduced above. These generators are attached to the boundary in the sense that vector fields ξ, χ, which on the boundary coincide together with their first derivatives, define generators differing by a constraint. Now computing the Poisson bracket between two generators we obtain (with a substantial effort) a result of the form

$$\{\mathcal{O}[\xi], \mathcal{O}[\psi]\} = \mathcal{O}[[\xi,\psi]_{SD}] + K[\xi,\psi] \tag{3.13}$$

where $K[\xi,\psi]$ is an a priori non vanishing central charge

$$\begin{aligned} K[\xi,\psi] &= \left([\xi,\psi]^{\|}_{SD}\Pi_\eta\eta - (\xi^\|\psi^{\|\prime} - \xi^{\|\prime}\psi^\|)\Pi_\sigma\sigma\right)_{|\partial\Sigma} \\ &+ \left((\xi^\|\psi^\perp - \psi^\|\xi^\perp)\left(\Pi_\eta\Pi_\sigma + V(\eta)\sigma\right) - \frac{\eta}{\sigma}([\xi,\psi]^\perp_{SD})'\right)_{|\partial\Sigma} \end{aligned} \tag{3.14}$$

and the bracket $[\,,\,]_{SD}$ is the well known surface deformation algebra for a two dimensional theory

$$[\xi,\psi]^\perp_{SD} = \xi^\|\psi^{\perp\prime} - \psi^\|\xi^{\perp\prime}, \qquad [\xi,\psi]^\|_{SD} = \frac{\xi^\perp\psi^{\perp\prime} - \psi^\perp\xi^{\perp\prime}}{\sigma^2} + \xi^\|\psi^{\|\prime} - \psi^\|\xi^{\|\prime} \tag{3.15}$$

We see that the possibility of a central charge is real. Indeed, as is well known [30], in three dimensional anti-de Sitter gravity the boundary at infinity gives a non vanishing $K[\xi,\psi]$. As we noted above, if there is a black hole and we use the Hamiltonian of a static external observer, then there is a contribution to (3.14) coming from the inner boundary at the horizon. But in the next section we shall show that if we impose the presence of a black hole, this contribution to the central charge actually vanishes.

4 The black hole and the near horizon symmetry

As far as we know, the presence of a black hole is always associated to the existence of an event horizon. Such an horizon is not a real, physical boundary of space-time, but from the viewpoint of an observer who chooses to stay permanently outside the black hole, we may treat it like a boundary. The horizon boundary conditions (3.9) then makes it possible to predict the exterior region alone, by giving initial data on a partial Cauchy surface extending from the horizon to spatial infinity, always external to the black hole. This is just the property of asymptotics predictability, a cornerstone of classical black hole physics.

If we analyze the behaviour of the Hamiltonian generators of our theory, we find that there are some transformations which change these boundary conditions and there are other transformations leaving them unchanged: these are the "gauge" transformations of the theory. The other ones change the black hole. We may fix $N^x = 0$ since on static solutions it can be globally removed by a change of coordinates. We find that $\Pi_{\eta|\partial\Sigma} = \Pi_{\sigma|\partial\Sigma} = 0$ and the horizon central charge (3.14) on such static solutions becomes

$$K[\xi,\psi] = \left((\xi^\|\psi^\perp - \psi^\|\xi^\perp)\left(\frac{\eta'}{\sigma}\right)' - \frac{\eta}{\sigma}([\xi,\psi]^\perp_{SD})'\right)_{\partial\Sigma} \tag{4.1}$$

We are interesting in studying the class of diffeomorphisms ξ that do not change the black hole, in particular we want that the boundary conditions (3.9) be preserved. So near the horizon at x_0 we ask for the following asymptotic behaviour of the lapse function

$$N^2 = O(x - x_0), \quad N'/\sigma = \kappa + O(x - x_0) \quad \text{as} \quad x \to x_0 \tag{4.2}$$

where the symbols $\phi = O(\psi)$ means that the ratio $\phi : \psi$ is bounded as $x \to x_0$. The linear dependence on $x - x_0$ is necessary to have a non zero surface gravity. More precisely, the regularity conditions (3.9) require the near horizon expansion

$$N^2 = (-g^{tt})^{-1} = 4\kappa^2 x_0(x - x_0) + \alpha(t)(x - x_0)^2 + O_3 \tag{4.3a}$$

$$\sigma^2 = g_{xx} = \frac{x_0}{x - x_0} + \Phi(x, t) \tag{4.3b}$$

$$\gamma_{tx} = \lambda(x - x_0) + O_3 \tag{4.3c}$$

where Φ and its first derivatives are regular at the horizon. The constant x_0 is arbitrary at this stage, but can be fixed in terms of the mass at infinity. For the Schwarzschild solution one finds of course $x_0 = 1/2\kappa$. A vector fields ξ preserving these conditions can at most change the metric according to the rules

$$\delta N^2 = \mathcal{L}_\xi N^2 = O(x - x_0), \qquad \mathcal{L}_\xi \sigma^2 = O(1), \qquad \mathcal{L}_\xi \gamma_{tx} = O(x - x_0)$$

and then we will have well defined observables $\mathcal{O}[\xi]$ with boundary charges $Q[\xi]$ (we restore $2G$ now)

$$Q[\xi] = \left.\frac{\eta\, \xi^{\perp\prime}}{2G\sigma}\right|_{\partial\Sigma} \tag{4.4}$$

These boundary observables (in the sense explained in Ref. [61]) are the Noether charges belonging to the diffeomorphism generated by ξ. If ξ is the Killing time translation symmetry then $\xi^\perp = N$ and the boundary conditions (3.9) gives $Q = Q_H = \kappa\eta_0/2G$ (cf. (2.10)).
We recall that the boundary conditions are essential to have well defined, i.e. differentiable, Hamiltonian generators $\mathcal{O}[\xi]$. The diffeomorphism which preserve the conditions (3.9) lead to well defined Hamiltonian generators. If we drop the condition $\delta\sigma^2 = O(1)$ or $\delta N^2 = O(x - x_0)$, the Hamiltonian generators are not differentiable, and consequently, not well defined as generators.
The regularity conditions being equivalent to the expansions (4.3a), (4.3b), (4.3c) near the bolt, an elementary computation gives the components of the diffeomorphism preserving generators as

$$\xi^x = -\kappa^{-1} N^2 \partial_t \xi^t + O(N^3), \qquad \xi^t = O(1) \tag{4.5}$$

$$\xi^x \frac{1}{(x - x_0)^2} - 2\partial_x \xi^x \frac{1}{x - x_0} = O(1) \tag{4.6}$$

Vector fields satisfying these equations and having limit on the horizon together with their first derivatives, have two important properties. First, given ξ_1, ξ_2 both satisfying (4.5), the Lie bracket also satisfies (4.5), so these vector fields close a Lie algebra.
The second property requires the surface deformation algebra [62], which has the form

$$[\xi_1, \xi_2]^\perp_{SD} = \xi_1^x \partial_x (N\xi_2^\tau) - \xi_2^x \partial_x (N\xi_1^\tau) \tag{4.7}$$

$$[\xi_1, \xi_2]^x_{SD} = \xi_1^x \partial_x \xi_2^x - \xi_2^x \partial_x \xi_1^x + N^3 \left(\xi_1^\tau \partial_x (N\xi_2^\tau) - \xi_2^\tau \partial_x (N\xi_1^\tau)\right) \tag{4.8}$$

Then, if ξ_1 and ξ_2 satisfy (4.5),

$$[\xi_1, \xi_2]_{SD} = [\xi_1, \xi_2] + O(N^3)$$

Requiring the vector fields ξ together with their first derivatives to have a limit on the horizon, the equations (4.5), (4.6) imply diffeomorphisms of the form

$$\xi^t = \chi_0 + \chi_1(x - x_0) + \sum_{k=2}^{\infty} a_k(t)(x - x_0)^k \tag{4.9a}$$

$$\xi^x = \sum_{k=1}^{\infty} b_k(t)(x - x_0)^{k+1} \tag{4.9b}$$

where the functions $b_k(t)$ are proportional to the time derivatives of the functions $a_k(t)$. These vectors form a Lie subalgebra of the Lie algebra of all vector fields. Using the asymptotic behaviour of ξ^a on the horizon it is a simple matter to show that $K[\xi_1, \xi_2]$, as given by (3.14), vanishes. Moreover the boundary observables (4.4) are all proportional to the Noether charge $Q_H = \kappa\eta_0/2G$, and form therefore a one-dimensional abelian algebra.

We recall that the Poisson bracket of two differentiable generators is a differentiable generator [63], so the observable $\mathcal{O}[\xi, \psi]_{SD}$ automatically includes the correct boundary terms up to a constant $K[\xi, \psi]$ which depends only on the asymptotic form of the vectors ξ, ψ. What happens here is that this constant actually vanishes for every choice of ξ and ψ satisfying our boundary conditions, and that the bracket $[\xi, \psi]_{SD}$ is a pure gauge, i.e. it has zero charge.

We also recall that the Hamiltonian generators are defined up to constants. Such constants are usually fixed by normalizing the Hamiltonians to given values in some background manifold. Our charges are normalized so that Q_H, given by (2.10), is the mass as measured at infinity. Sometimes other normalizations are proposed. In [64], for example, normalized to zero values are the horizon charges of the generators of the enveloping algebra $\mathcal{O}[\hat{\xi}_{-1}]$, $\mathcal{O}[\hat{\xi}_0]$, $\mathcal{O}[\hat{\xi}_1]$, and after this a central charge $K[\hat{\xi}_{-2}, \hat{\xi}_2]$ is found. We stress that the normalization cannot determine whether a central charge is present or not, it can only affect its actual value.

The only admissible way to find a non zero central charge on the horizon is to stretch the horizon and consider diffeomorphisms that do not change the boundary value of the Hamiltonian on this "stretched" horizon [65, 64]. This calculation leads to nonzero charges and to a nonzero central charge when the stretched horizon tends to the physical horizon. In this cases the diffeomorphisms are not defined in the limit. This seems to be unsatisfactory for the definition of a conformal algebra on the horizon. Finally, there remains the possibility that the symmetries involved in the boundary observables are the transverse angular vectors that we lost because of our dimensional reduction. This seems unlikely to us, since these diffeomorphisms should preserve the area of the horizon and there are no central extensions for the area preserving diffeomorphisms on the sphere [66].

5 Another route to a central charge

The fact that no central charge can emerge from boundary observables does not prove the absolute absence of a classical central charge in $2D$ theories of the kind we have discussed. There remains the possibility of a charge emerging from the bulk observables, as a Schwinger term in the constraint algebra. In fact, it is well known that in two dimensions special conditions affect the constraint algebra. One fact is that the Hamiltonian generators may not vanish on shell, but the equations of motion can still be generally covariant. This is possible because in two dimensions the structure coefficients of the constraint algebra (not really an algebra)

$$\{H_\mu(x), H_\nu(x')\} = \int dx'' \, K^\rho_{\mu\nu}(x, x'; x'')H_\rho(x'')$$

can be made independent on the canonical variables. With generators $H_\mu(x)$ not constrained to vanish, a central term can be consistently added to the left hand side of this equation.

In our case however, we started with a covariant theory and this has the consequence that the generators must all vanish on shell, as is well known. No classical Schwinger terms are then possible in the constraint algebra. We think this is one reason why people searched for a central extension in the algebra of boundary observables, rather than among constraints.
We now present an argument, similar in spirit to Verlinde's rewriting of FRW equations as an entropy formula [67], which suggest the presence of a classical central charge. The Hamiltonians (3.10) were deduced for a singular foliation where all surfaces intersect at the bifurcation point of the event horizon. They consist of a term at infinity, giving the energy, and a term at the horizon. As we explained, this term is what fixes the disk topology of the black hole. Suppose we write this Hamiltonian in the form given by a conformal field theory on the disk[10]

$$\kappa^{-1}H = L_0 + \bar{L}_0 - \frac{c+\bar{c}}{24} \tag{5.1}$$

where the first two terms correspond to the boundary at infinity (the energy) and the last two to the boundary at the horizon. For a non singular foliation, with no intersections at the horizon, there would be no such terms in the Hamiltonian. This would correspond to the Hamiltonian of a conformal field theory as given on the cylinder. Let we assume a symmetric contribution of left/right moving modes, i.e. $L_0 = \bar{L}_0$ and $c = \bar{c}$ (non rotating black holes). The mass at infinity is $\kappa A/4\pi G$ so from (5.1) we get[11]

$$L_0 = \frac{A}{8\pi G} = \frac{\eta_0}{2G} \tag{5.2}$$

The Noether charge at the horizon is $\kappa\eta_0/2G$, so again from (5.1) we get

$$c = \frac{6\eta_0}{G} = \frac{3A}{2\pi G} \tag{5.3}$$

Since L_0 is comparable to c, the entropy should be [68]

$$S = 2\pi\sqrt{\frac{c}{6}\left(L_0 - \frac{c}{24}\right)} = \frac{\pi\eta_0}{G} = \frac{A}{4G} \tag{5.4}$$

Eq. (5.3) is the memory of dimensional reduction. It can be represented pictorially as follows: since the actual geometry is the disk times a sphere with area A, we have in effect a $2D$ theory attached at each Planck-sized cell on the sphere. The central charge is additive, whence the result.
There is also another interpretation of (5.1), which is that it can be viewed as an Euclidean conformal field theory on the horizon. In fact, as shown in [69], it is possible to define a free quantum field theory on the future and past horizons in terms of $2D$ conformal field theories. These $2D$ fields are: in the holomorphic case $\hat{\phi}(z) = \sum_n z^n a_n/n$ and in the anti-holomorphic case $\hat{\phi}(\bar{z}) = \sum_n \bar{z}^n b_n/n$. Here $z = \exp(-i\arctan(\kappa\, it + \kappa\, r_*))$, where r_* is the tortoise coordinate, and t is the complexified time variable on the horizon as seen by an observer at spatial infinity[12]. As usual, the Virasoro generators are the Fourier component of the Euclidean stress tensor $T(z) = \sum_{n\in\mathbb{Z}/\{0\}} z^{-n-2}L_n$, $T(\bar{z}) = \sum_{n\in\mathbb{Z}/\{0\}} \bar{z}^{-n-2}\bar{L}_n$. Since the central charge of this theory is different from 1, it can be naturally interpreted as a two dimensional linear dilaton conformal field theory whose stress-energy tensor has the following components [70]

$$T_{zz} = T(z) \quad = \quad -:\partial_z\hat{\phi}(z)\,\partial_z\hat{\phi}(z): + V\,\partial_z^2\,\hat{\phi}(z), \tag{5.5}$$
$$T_{\bar{z}\bar{z}} = T(\bar{z}) \quad = \quad -:\partial_{\bar{z}}\hat{\phi}\,(\bar{z})\,\partial_{\bar{z}}\hat{\phi}(\bar{z}): + V\,\partial_{\bar{z}}^2\,\hat{\phi}(\bar{z}). \tag{5.6}$$

[10] The factor $\kappa^{-1} = \beta/2\pi$ means that scaling the period of euclidean time from 2π to β changes the Hamiltonian as written.
[11] Recall that the dilaton at the horizon is $\eta_0 = A/4\pi$.
[12] See [69] for details.

with $V^2 = \eta_0/G = A/(4\pi G)$. The corresponding central charge c is equal to $1 + 6\eta_0/G$. In the semi-classical regime $\eta_0/G >> 1$, so this does not affect the computation of the entropy in (5.4). Moreover, as shown in [69], the field $\hat{\phi}(z)$ can be interpreted as a free field in the bulk of a Rindler spacetime.

We think this qualitative argument is suggestive enough to deserve a more detailed investigation. We note that with the usual normalization of the action that is used in the JT model of $2D$ gravity, corresponding here to $G = 1/2$, the central charge $c = 12\eta_0$ is what has been found in several researches on the JT model with anti-de Sitter boundary conditions [71, 72, 73, 74]. The present argument suggests that the central charge is universal and independent on the form of the dilaton potential $V(\eta)$.

In the next section we abandon the search for a central charge, and present a very tentative model suitable to describe the quantum hairs. Our exercise follows a suggestion made in [61].

6 A covariant Action

The theory of relativity in a domain without boundary is invariant under diffeomorphisms, so diffeomorphic metrics will describe the same physics. In this sense the diffeomorphisms are like gauge transformations. The situation is totally different in the presence of a boundary, for in this case the gauge group is smaller. It contains only transformations which do not change the boundary value of the action. Technically speaking, this means that the generators of these transformations are the Hamiltonians corresponding to prescribed boundary conditions.

If we insist to consider the horizon as a boundary we necessarily lose part of the gauge group, and there will be gauge transformations which are not symmetries of the black hole. But we expect physics to be invariant under the full gauge group. There is a way to restore the gauge invariance of such systems [61], and this is adding suitable boundary field to the action, possibly coupled with the bulk fields.

We have analyzed the generators of the permitted transformations in Sec. [3]. Some of them are pure gauge transformation ($N = 0$, $N' = 0$ and $N^x = 0$ on the boundary). In this case the Hamiltonian generators $H[N, N^x]$ are like "Gauss laws", and the commutator of such Hamiltonians vanishes on shell. There are other generators that commute with the constraints, but they are not zero on shell, they are observables. To restore the gauge invariance we will add new external fields living on the horizon. We take the gravitational part of the action in the form

$$I = C \int_\Sigma |g|^{1/2} \left(\frac{1}{2}\eta \mathcal{R}[g] + V(\eta) \right) d^2x \tag{6.1}$$

with $V(\eta) = \frac{1}{\sqrt{\eta}}$ in the case of an asymptotically flat four dimensional theory. The variation of this action produces the boundary terms we discussed above. Let us consider for simplicity only the boundary term of a diagonal two dimensional metric, $(-N^2, \sigma^2)$, with fixed horizon at $x = x_0$, so $N(x_0) = 0$. Under the small change

$$N \to N + \delta N, \qquad \sigma \to \sigma + \delta\sigma, \qquad \eta \to \eta + \delta\eta \tag{6.2}$$

new boundary terms will appear in the action variation, making it not invariant

$$\delta I = \cdots + \int dt \left[-N\delta\left(\frac{\eta'}{\sigma}\right) + \left(\frac{N'}{\sigma}\right)\delta\eta - (N^x\Pi_\eta)\delta\eta + (N^x\sigma)\delta\Pi_\sigma \right]_{\partial\Sigma} \tag{6.3}$$

Only diffeomorphisms not bringing boundary terms are allowed as invariances of the action, but the gauge group should be larger. To obtain a gauge invariant action while keeping the boundary, we add the following extra fields ϕ_1, ϕ_2, ϕ_3 on the boundary, and assume the transformation rules

$$\phi_1 \to \phi_1 - \delta\frac{\eta'}{\sigma}_{|0} \qquad \phi_2 \to \phi_2 - \delta\eta_{|0} \qquad \phi_3 \to \phi_3 - \delta(\Pi_\sigma)_{|0} \tag{6.4}$$

Then adding the term

$$I_A = \int dt \left(N\phi_1 - \left(\frac{N'}{\sigma} - N^x \Pi_\eta \right) \phi_2 - N^x \sigma \phi_3 \right)_{x \to x_0} \tag{6.5}$$

the action $I + I_A$ becomes gauge invariant under the full diffeomorphism group. At this level the new fields have no dynamics. But nothing forbid us to add a gauge invariant dynamical part in the action with this covariant derivative

$$D\phi_1 = \partial\phi_1 + \partial \left(\frac{\eta'}{\sigma} \right)_{|0} \qquad D\phi_2 = \partial\phi_2 + \partial\eta_{|0} \qquad D\phi_3 = \partial\phi_3 + \partial\Pi_{\sigma\,|0} \tag{6.6}$$

so we may add the kinetic term

$$I_D = \int \Lambda_{AB} D\phi^A D\phi^B \tag{6.7}$$

for some field independent matrix Λ_{AB}. The new action $I + I_A + I_D$ is gauge invariant under all diffeomorphisms. It is interesting to note that the boundary part of the action does not depend on the form of the dilaton potential.

Now the exercise would be to do the statistical mechanics of the boundary fields. The strategy we wish to follow is considering the gravitational part of the action like that of a reservoir, and the part with the fields ϕ_A like the actual system. We observe that if we are interested in transformations made at fixed temperature κ, the interaction is particularly simple. In fact it becomes $I_A = \int (N\phi_1 - \kappa\phi_2)$, but the first part is small enough that we may drop it.

Then the action of this simplified model reads

$$I_D + I_A = \int dt \left(\Lambda \, D\phi D\phi - \kappa\phi \right)_{x \to 0} \tag{6.8}$$

where now ϕ stands for the field ϕ_2 described above. For the statistical mechanics of this model we need the Hamiltonian, $H_B = \Pi_\phi^2/\Lambda + \kappa\phi$, derived from (6.8). One may notice that it describes a freely falling particle with configuration variable ϕ in a theory with linear potential. Moreover, since ϕ can take only positive values, we have to introduce an infinite barrier at $\phi = 0$ in the potential of our model. As just pointed out ϕ is not a field but a quantum variable of our system, therefore we are dealing with a problem in ordinary quantum mechanics.

The Hamiltonian H_B has a discrete non degenerate spectrum. Though we lack an analytic expression for the energy eigenvalues of H_B, it is possible to estimate their values in the high energy limit. The wave function that solves the eigenvalue equation, $H\,\psi = E\,\psi$, reads

$$\psi(x) = A\Phi \left(\kappa^{1/3} \, \Lambda^{2/3} \left(x - \frac{E}{\kappa} \right) \right), \tag{6.9}$$

where $\Phi(x)$ is an appropriate Airy function. In particular, in the high energy limit the energy levels are proportional to $n^{2/3}$. To compute the partition function we also need to consider the density of states

$$\rho(E) = \frac{\Lambda}{\kappa\pi} \sqrt{E}. \tag{6.10}$$

Assuming that the black hole is described by several non interacting bosons forming a canonical system, we can do the statistical mechanics of this model.

First we fix the constant Λ in (6.10) so that the expected energy in a canonical ensemble with temperature $\kappa/2\pi$ and density of states (6.10), be equal to the mass of the black hole, $M = \kappa\,\eta_0$. This gives

$$\Lambda = \frac{16}{3}\frac{\pi^{5/2}}{\zeta(5/2)}\,\eta_0\,\sqrt{\beta} \tag{6.11}$$

where $\zeta(5/2) \simeq 1.3414$ is the Riemann zeta function at $5/2$, and $\beta = 2\pi\kappa^{-1}$ is the inverse of the Hawking temperature. With this curious choice of Λ the canonical partition function is

$$\log \mathcal{Z} \simeq -\int_0^\infty \log\left(1 - e^{-\beta E}\right)\rho(E)\,dE = \frac{4}{3}\pi\eta_0, \tag{6.12}$$

and

$$\langle E\rangle \simeq \int_0^\infty \frac{E\,\rho(E)\,dE}{e^{\beta E} - 1} = \frac{2\pi\eta_0}{\beta} = M \tag{6.13}$$

Moreover, the statistical entropy becomes

$$S = -\beta^2 \frac{\partial}{\partial\beta}\frac{\log \mathcal{Z}}{\beta} = \frac{4\pi}{3}\eta_0 \tag{6.14}$$

which is a little higher (4/3) than the Bekenstein-Hawking value, but satisfies the area law. Perhaps the right factor can be restored by considering the other fields ϕ_1 and ϕ_3, but our point was not so much to be exact with the entropy. Rather, we wanted to show that the boundary fields, as suggested by the Hamiltonian description of a diffeomorphism invariant theory, could really do the right job.

7 Conclusion

It is interesting to note that the analysis of the Hamiltonian generators near the horizon does not reproduce the microstates of black holes. On the other hand, the relevant observables (perhaps not all) seem localized precisely at the horizon, but only for those observers staying permanently outside the black hole and detecting a static field. Others, Kruskal-like observers do not need boundary observables to give a Hamiltonian description of space-time. If one tries to effectively describe the black hole, he should realize at a quantum level a connection between the inner and the outer part of the horizon, through the addition of boundary fields. These are necessary to restore the full invariance group, broken by the choice of boundary conditions. We have shown that the statistical description of these fields is at least compatible with the thermodynamics description of the black hole.

Acknowledgments

We thanks Valter Moretti for useful discussions and technical suggestions.

References

[1] R. M. Wald, Living Rev. Relativity, **4**, 6 (2001).

[2] G. W. Gibbons and S. W. Hawking, Phys. Rev. **D 15**, 2738 (1977).

[3] T. Jacobson and R. Parentani, [arXiv:gr-qc/0302099].

[4] P. C. Davies, Class. Quant. Grav. **5**, 1349 (1988).

[5] G. 't Hooft, Nucl. Phys. **B 256** 727-745 (1985).

[6] J. D. Bekenstein, Lett. Nuovo Cim. **4**, 737 (1972).

[7] J. D. Bekenstein, Prodeedings of the Eight Marcel Grossmann Meeting, T. Piran and R. Ruffini, eds. (World Scientific Singapore 1999), [arXiv:gr-qc/9710076].

[8] A. Barvinskii and G. Kunstatter, Phys. Lett. **B 389**, 231 (1969).

[9] V.F. Mukhanov, Pis'ma Zh. Eksp. Teor. Fiz. **44**, 50 (1986), (JETP Lett. **44**, 63 (1986)).

[10] I. Kogan,Pis'ma Zh. Eksp. Teor. Fiz. **44**, 209(1986), (JETP Lett. **44**, 267 (1986)).

[11] P.O. Mazur, Phys. Rev. Lett. **57**, 929 (1986) and **59**, 2380 (1987).

[12] J. García-Bellido, Report SU-ITP-93-4, [arXiv:hep-th/9302127].

[13] U.H. Danielsen and M. Schiffer, Phys. Rev. **D 48**, 4779 (1993).

[14] Y. Peleg, Report BRX-TH-350, [arXiv:hep-th/9307057].

[15] M. Maggiore, Nucl. Phys. **B 429**, 205 (1994).

[16] C.O. Lousto, Phys. Rev. **D 51**, 1733 (1995).

[17] J.D. Bekenstein and V.F. Mukhanov, Phys. Lett. **B360**, 7 (1995).

[18] H.A. Kastrup, Phys. Lett. **B 385**, 75 (1996).

[19] H. A. Kastrup, Phys. Lett. **B 413**, 267 (1997).

[20] V. Berezin, Phys. Rev. **D 55**, 2139 (1997).

[21] C. Vaz, Phys. Rev. **D 61**, 064017 (2000).

[22] C. Vaz, L. Witten, Phys. Rev. **D 63**, 024008 (2001).

[23] S. Hod, Phys. Rev. Lett. **81**, 4293 (1998).

[24] G. Kunstatter, Phys. Rev. Lett. **90**, 161301 (2003).

[25] O. Dreyer, Phys. Rev. Lett. **90**, 081301 (2003).

[26] J. York, Phys. Rev. **D 28** 2929 (1983).

[27] L. Bombelli, R. K. Koul, J. Lee and R. Sorkin, Phys. Rev. **D 34**, 373 (1986).

[28] T. Jacobson, [arXiv:gr-qc/9404039].

[29] L. Susskind and J. Uglum, Phys. Rev. **D 50**, 2700 (1994).

[30] J.D.Brown and M.Henneaux, Commun. Math. Phys. **104**, 207 (1986).

[31] A. Strominger, JHEP**9802**, 009 (1998).

[32] M. Bañados, C. Teitelboim and J. Zanelli, Phys. Rev. Lett. **69**, 1849 (1992).

[33] J. M. Maldacena, Adv. Theor. Math. Phys. **2**, 231 (1998).

[34] A. Strominger and C. Vafa, Phys. Lett. **B379**, 99 (1996).

[35] C. G. Callan and J. M. Maldacena, Nucl. Phys. **B472**, 591 (1996).

[36] S. S. Gubser, I. R. Klebanov and A. W. Peet, Phys. Rev. **D54**, 3915-3919 (1996).

[37] S. N. Solodukhin, Phys. Lett. **B 454**, 213 (1999).

[38] S. Carlip, Phys. Rev. Lett. **82**, 2828 (1999).

[39] F. Lin and Y. Wu, Phys. Lett. **B453**, 222 (1999).

[40] M. Park and J. Ho, Phys. Rev. Lett. **83**, 5595 (1999).

[41] M. Park and J. H. Yee, Phys. Rev. **D 61**, 088501 (2000).

[42] V. O. Soloviev, Phys. Rev. **D61**, 027502 (2000).

[43] M. Hotta, K. Sasaki and T. Sasaki, Class. Quant. Grav. **18**, W1823 (2001).

[44] J. Koga, Phys. Rev. **D64**, 124012 (2001).

[45] R. Jackiw, *Liouville Field Theory: a Two-Dimensional Model for Gravity?*, C. Teitelboim, *The Hamiltonian Structure of two-dimensional Space-Time*, in Quantum Theory of Gravity, essays in honor of B. De Witt, edited by S. Christensen (1984).

[46] S. B. Giddins and A. Strominger, Phy. Rev. **D47** (1993), 2454.

[47] R. Brustein, Phys. Rev. Lett. **86**, 576 (2001).

[48] S. D. Odintsov ans I Shapiro, Phys. Lett. **B263** (1991), 183;

[49] S. D. Odintsov ans I Shapiro, Int. J. Mod. Phys. **D1** (1992), 571.

[50] J. G. Russo and A. A. Tseytlin, Nucl. Phys. **B382**, 259 (1992).

[51] C. Callan, S. Giddings, J. Harvey, A. Strominger Phys. Rev. **D45**, 1005 (1992).

[52] R. M. Wald, Phys. Rev. **D48**, R3427 (1993).

[53] V. Iyer and R. M. Wald, Phys. Rev. **D50**, 846 (1994).

[54] K. K. Kuchar, J. D. Romano and M. Varadarajan, Phys. Rev. **D55** (1997), 795.

[55] T. Strobl, Habilitation thesis RWTH Aachen, [arXiv: hep-th:0011240].

[56] S. Nojiri and S.D. Odintsov, Int. J. Mod. Phys. **A16** (2001), 1015.

[57] D. Grumiller, W. Kummer, D. V. Vassilevich Phys. Rept. **369** (2002), 327-430

[58] J. D. Brown, S. R. Lau, J. W. York, [arXiv:gr-qc/0010024].

[59] T. Regge and C. Teitelboim, Ann. Phys. **88**, 286 (1974).

[60] J. D. Brown, G. L. Comer, E. A. Martinec, J. Melmed, B. F. Whiting and J. W. York Jr, Class. Quant. Grav. **7**, 1433 (1990).

[61] A.P. Balachandran, L. Chandar, Arshad Momen, Nuc. Phys. **B 461**, 581 (1996).

[62] C. Teitelboim, Ann. Phys. **79**, 542 (1973).

[63] J. D. Brown, M. Henneaux, J. Math. Phys. **27**(2), 489 (1986).

[64] S. Silva, Class. Quant. Grav. **19**, 3947 (2002).

[65] S. Carlip, Phys. Rev. Lett. **88**, 241301 (2002). [arXiv:gr-qc/020300].

[66] I. Bars, C. N. Pope and E. Sezgin, Phys. Lett. **B210**, 85 (1988).

[67] E. Verlinde, [arXiv:hep-th/0008140].

[68] J. A. Cardy, Nuc. Phys. **B 270**, 186 (1986).

[69] V. Moretti, N. Pinamonti, to be publilshed on Jour. Math. Phys. [arXiv:hep-th/0307021].

[70] J. Polchinski, *String Theory*, Vol. I, Cambridge University Press (1998).

[71] M. Cadoni, S. Mignemi, Nucl. Phys. **B 557**, 165 (1999).

[72] M. Cadoni, P. Carta, D. Klemm and S. Mignemi, Phys. Rev. **D63**, 125021 (2001).

[73] M. Caldarelli, G. Catelani, L. Vanzo, JHEP**0010**, 005 (2000).

[74] G. Catelani, L. Vanzo, [arXiv:hep-th/0009186].